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The Validity of Generalized Modal Syllogisms with the Generalized Quantifiers in Square{*most*}

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The Validity of Generalized Modal Syllogisms with the Generalized Quantifiers in Square {*most*}

Siyi Yu ^α & Xiaojun Zhang ^σ

Abstract- Due to the large number of generalized quantifiers in the English language, this paper only studies the fragment of generalized modal syllogistic that contains the quantifiers in Square{all} and Square{most}. On the basis of generalized quantifier theory, possible-world semantics, and set theory, this paper shows that there are reducible relations between/among the generalized modal syllogism $\text{EM}\diamond\text{O-3}$ and at least the other 29 valid generalized modal syllogisms. This method can also be used to study syllogisms with other generalized quantifiers. The results obtained by means of formal deductive method have not only consistency, but also theoretical value for the development of inference theory in artificial intelligence.

Keywords: *generalized modal syllogisms; reducibility; modality; validity.*

I. INTRODUCTION

Syllogism is one of the significant forms of reasoning in natural language and human thinking. There are various kinds of syllogisms, such as Aristotelian syllogisms (Patzig, 1969; Long, 2023; Hui, 2023), Aristotelian modal syllogisms (Johnson, 2004; Łukasiewicz, 1957; Cheng and Xiaojun, 2023), generalized syllogisms (Murinová and Novák, 2012; Xiaojun and Baoxiang, 2021; Endrullis and Moss, 2015), and generalized modal syllogisms (Jing and Xiaojun, 2023).

Although many generalized modal syllogisms exist in natural language, there is little literature on their reducibilities. Therefore, this paper mainly focuses on them. The four Aristotelian quantifiers (that is, *not all*, *all*, *some* and *no*) constitute Square{all}. And '*most*' and its three negative (i.e. inner, outer and dual), *fewer than half of the*, *at most half of the*, and *at least half of the*, form Square{most}. The generalized modal syllogisms studied in this paper only involve the quantifiers in Square{all} and Square{most}.

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II. PRELIMINARIES

In this paper, let w , v and z be the lexical variables, which are elements in the set W , V and Z respectively, D be the domain of lexical variables, $|W|$ the cardinality of the set W , and m , n , s and t propositional variables. Q stands for any generalized quantifiers, $\neg Q$ and $Q\neg$ for the outer and inner negative quantifier of Q respectively. The generalized modal syllogisms discussed in this paper comprise the following sentences as follows: ‘all w s are v s’, ‘no w s are v s’, ‘some w s are v s’, ‘not all w s are v s’, ‘most w s are v s’, ‘fewer than half of the w s are v s’, ‘at most half of the w s are v s’, and ‘at least half of the w s are v s’. They can be denoted as: $all(w, v)$, $no(w, v)$, $some(w, v)$, $not\ all(w, v)$, $most(w, v)$, $fewer\ than\ half\ of\ the(w, v)$, $at\ most\ half\ of\ the(w, v)$, $at\ least\ half\ of\ the(w, v)$, and are respectively abbreviated as Proposition A , E , I , O , M , F , H and S .

A non-trivial generalized modal syllogism includes at least one and at most three non-overlapping modalities (possible modality (\Diamond) or necessary modality (\Box)) and non-trivial generalized quantifiers, such as the quantifiers in Square-most”.

Example 1:

Major premise: No grapes are necessarily blueberries.

Minor premise: Most grapes are purple fruits.

Conclusion: Not all purple fruits are possibly blueberries.

Let w be the lexical variable for a blueberry in the domain, v be the lexical variable for a grape in the domain, and z be the lexical variable for a purple fruit in the domain. Then the syllogism in example 1 can be formalized as: $\Box no(v, w) \wedge most(v, z) \rightarrow \Diamond not\ all(z, w)$, which abbreviated as $\Box EM\Diamond O-3$.

According to generalized quantifier theory, set theory (Halmos, 1974) and possible world semantics (Chellas, 1980), the truth value definitions of sentences with quantification, relevant facts and rules used in the paper are as follows:

Definition 1 (truth value definitions):

- (1.1) $all(w, v)$ is true when and only when $W \subseteq V$ is true in all real worlds.
- (1.2) $no(w, v)$ is true when and only when $W \cap V = \emptyset$ is true in all real worlds.
- (1.3) $some(w, v)$ is true when and only when $W \cap V \neq \emptyset$ is true in all real worlds.
- (1.4) $not\ all(w, v)$ is true when and only when $W \not\subseteq V$ is true in all real worlds.
- (1.5) $most(w, v)$ is true when and only when $|W \cap V| > 0.5|W|$ is true in all real worlds.
- (1.6) $\Box all(w, v)$ is true when and only when $W \subseteq V$ is true in all possible worlds.
- (1.7) $\Diamond all(w, v)$ is true when and only when $W \subseteq V$ is true in some possible worlds.
- (1.8) $\Box no(w, v)$ is true when and only when $W \cap V = \emptyset$ is true in all possible worlds.
- (1.9) $\Diamond no(w, v)$ is true when and only when $W \cap V = \emptyset$ is true in some possible worlds.
- (1.10) $\Box some(w, v)$ is true when and only when $W \cap V \neq \emptyset$ is true in all possible worlds.

(1.11) $\Diamond some(w, v)$ is true when and only when $W \cap V \neq \emptyset$ is true in some possible worlds.

(1.12) $\Box not\ all(w, v)$ is true when and only when $W \not\subseteq V$ is true in all possible worlds.

(1.13) $\Diamond not\ all(w, v)$ is true when and only when $W \not\subseteq V$ is true in some possible worlds.

(1.14) $\Box most(w, v)$ is true when and only when $|W \cap V| > 0.5|W|$ is true in all possible worlds.

(1.15) $\Diamond most(w, v)$ is true when and only when $|W \cap V| > 0.5|W|$ is true in some possible worlds.

Definition 2 (inner negation): $Q\neg(w, v) =_{\text{def}} Q(w, D-v)$.

Definition 3 (outer negation): $\neg Q(w, v) =_{\text{def}}$ It is not that $Q(w, v)$.

Fact 1 (inner negation):

(1.1) $\vdash all(w, v) \leftrightarrow no\neg(w, v)$;

(1.2) $\vdash no(w, v) \leftrightarrow all\neg(w, v)$;

(1.3) $\vdash some(w, v) \leftrightarrow not\ all\neg(w, v)$;

(1.4) $\vdash not\ all(w, v) \leftrightarrow some\neg(w, v)$;

(1.5) $\vdash fewer\ than\ half\ of\ the(w, v) \leftrightarrow most\neg(w, v)$;

(1.6) $\vdash most(w, v) \leftrightarrow fewer\ than\ half\ of\ the\neg(w, v)$;

(1.7) $\vdash at\ most\ half\ of\ the(w, v) \leftrightarrow at\ least\ half\ of\ the\neg(w, v)$;

(1.8) $\vdash at\ least\ half\ of\ the(w, v) \leftrightarrow at\ most\ half\ of\ the\neg(w, v)$.

Fact 2 (outer negation):

(2.1) $\vdash \neg not\ all(w, v) \leftrightarrow all(w, v)$;

(2.2) $\vdash \neg all(w, v) \leftrightarrow not\ all(w, v)$;

(2.3) $\vdash \neg no(w, v) \leftrightarrow some(w, v)$;

(2.4) $\vdash \neg some(w, v) \leftrightarrow no(w, v)$;

(2.5) $\vdash \neg most(w, v) \leftrightarrow at\ most\ half\ of\ the(w, v)$;

(2.6) $\vdash \neg at\ most\ half\ of\ the(w, v) \leftrightarrow most(w, v)$;

(2.7) $\vdash \neg fewer\ than\ half\ of\ the(w, v) \leftrightarrow at\ least\ half\ of\ the(w, v)$;

(2.8) $\vdash \neg at\ least\ half\ of\ the(w, v) \leftrightarrow fewer\ than\ half\ of\ the(w, v)$.

Fact 3 (dual):

(3.1) $\vdash \neg \Box Q(w, v) \leftrightarrow \Diamond \neg Q(w, v)$;

(3.2) $\vdash \neg \Diamond Q(w, v) \leftrightarrow \Box \neg Q(w, v)$.

Fact 4 (symmetry):

(4.1) $\vdash some(w, v) \leftrightarrow some(v, w)$;

(4.2) $\vdash no(w, v) \leftrightarrow no(v, w)$.

Fact 5 (subordination):

$$(5.1) \vdash \Box Q(w, v) \rightarrow Q(w, v);$$

$$(5.2) \vdash \Box Q(w, v) \rightarrow \Diamond Q(w, v);$$

$$(5.3) \vdash Q(w, v) \rightarrow \Diamond Q(w, v);$$

$$(5.4) \vdash all(w, v) \rightarrow some(w, v);$$

$$(5.5) \vdash no(w, v) \rightarrow not\ all(w, v).$$

Rule 1 (subsequent weakening): If $\vdash(m \wedge n \rightarrow s)$ and $\vdash(s \rightarrow t)$, then $\vdash(m \wedge n \rightarrow t)$.

Rule 2 (anti-syllogism): If $\vdash(m \wedge n \rightarrow s)$, then $\vdash(\neg s \wedge m \rightarrow \neg n)$ or $\vdash(\neg s \wedge n \rightarrow \neg m)$.

III. THE VALIDITY OF THE SYLLOGISM $\Box EM \Diamond O-3$

In order to discuss the reducibility of generalized modal syllogisms based on the syllogism $\Box EM \Diamond O-3$, it is necessary to prove the validity of the syllogism $\Box EM \Diamond O-3$.

Theorem 1 ($\Box EM \Diamond O-3$): The generalized modal syllogism $\Box no(v, w) \wedge most(v, z) \rightarrow \Diamond not\ all(z, w)$ is valid.

Proof: According to Example 1, $\Box EM \Diamond O-3$ is the abbreviation of the syllogism $\Box no(v, w) \wedge most(v, z) \rightarrow \Diamond not\ all(z, w)$. Suppose that $\Box no(v, w)$ and $most(v, z)$ are true, then in virtue of Definition (1.8), $\Box no(v, w)$ is true when and only when $V \cap W = \emptyset$ is true in all possible worlds. Similarly, in line with Definition (1.5), $most(v, z)$ is true when and only when $|V \cap Z| > 0.5|V|$ is true in all real worlds. Real worlds are elements in the set of all possible worlds. Thus, it is easily seen that $V \cap W = \emptyset$ and $|V \cap Z| > 0.5|V|$ are true in some possible worlds. Then, it is clear that $Z \not\subseteq W$ is true in some possible worlds. $\Diamond not\ all(z, w)$ is true in terms of Definition (1.13). The above proves that the syllogism $\Box no(v, w) \wedge most(v, z) \rightarrow \Diamond not\ all(z, w)$ is valid.

IV. THE OTHER GENERALIZED MODAL SYLLOGISMS DERIVED FROM $\Box EM \Diamond O-3$

Theorem 1 states that $\Box EM \Diamond O-3$ is valid, and ' $\Box EM \Diamond O-3 \rightarrow \Box EM \Diamond O-4$ ' in Theorem 2(1) expresses that the validity of syllogism $\Box EM \Diamond O-4$ is deduced from that of syllogism $\Box EM \Diamond O-3$. That is to show that there are reducible relations between these two syllogisms, and the others are similar.

Theorem 2: There are at least the following 29 valid generalized modal syllogisms obtained from $\Box EM \Diamond O-3$:

$$(1) \Box EM \Diamond O-3 \rightarrow \Box EM \Diamond O-4$$

$$(2) \Box EM \Diamond O-3 \rightarrow \Box A \Box EH-2$$

$$(3) \Box EM \Diamond O-3 \rightarrow \Box AM \Diamond I-1$$

$$(4) \Box EM \Diamond O-3 \rightarrow \Box AM \Diamond I-3$$

- (5) $\Box EM\Diamond O-3 \rightarrow \Box EM\Diamond O-4 \rightarrow \Box A\Box EH-4$
- (6) $\Box EM\Diamond O-3 \rightarrow \Box EM\Diamond O-4 \rightarrow M\Box A\Diamond I-4$
- (7) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box E\Box AH-2$
- (8) $\Box EM\Diamond O-3 \rightarrow \Box AM\Diamond I-1 \rightarrow \Box EM\Diamond O-1$
- (9) $\Box EM\Diamond O-3 \rightarrow \Box AM\Diamond I-3 \rightarrow M\Box A\Diamond I-3$
- (10) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box E\Box AH-2 \rightarrow \Box E\Box AH-1$
- (11) $\Box EM\Diamond O-3 \rightarrow \Box AM\Diamond I-1 \rightarrow \Box EM\Diamond O-1 \rightarrow \Box EM\Diamond O-2$
- (12) $\Box EM\Diamond O-3 \rightarrow \Box AM\Diamond I-3 \rightarrow M\Box A\Diamond I-3 \rightarrow F\Box A\Diamond O-3$
- (13) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box E\Box AH-2 \rightarrow \Box E\Box AH-1 \rightarrow \Box A\Box AS-1$
- (14) $\Box EM\Diamond O-3 \rightarrow \Box AM\Diamond I-1 \rightarrow \Box EM\Diamond O-1 \rightarrow \Box EM\Diamond O-2 \rightarrow \Box AF\Diamond O-2$
- (15) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box A\Box E\Diamond H-2$
- (16) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box A\Box E\Diamond H-2 \rightarrow \Box A\Box E\Diamond H-4$
- (17) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box A\Box E\Diamond H-2 \rightarrow \Box A\Box M\Diamond I-1$
- (18) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box A\Box E\Diamond H-2 \rightarrow \Box E\Box M\Diamond O-3$
- (19) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box A\Box E\Diamond H-2 \rightarrow \Box E\Box A\Diamond H-2$
- (20) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box A\Box E\Diamond H-2 \rightarrow \Box A\Box E\Diamond H-4 \rightarrow \Box M\Box A\Diamond I-4$
- (21) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box A\Box E\Diamond H-2 \rightarrow \Box A\Box E\Diamond H-4 \rightarrow \Box E\Box M\Diamond O-4$
- (22) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box A\Box E\Diamond H-2 \rightarrow \Box A\Box M\Diamond I-1 \rightarrow \Box E\Box M\Diamond O-1$
- (23) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box A\Box E\Diamond H-2 \rightarrow \Box E\Box M\Diamond O-3 \rightarrow \Box A\Box M\Diamond I-3$
- (24) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box A\Box E\Diamond H-2 \rightarrow \Box E\Box A\Diamond H-2 \rightarrow \Box E\Box A\Diamond H-1$
- (25) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box A\Box E\Diamond H-2 \rightarrow \Box A\Box M\Diamond I-1 \rightarrow \Box E\Box M\Diamond O-1$
 $\rightarrow \Box E\Box M\Diamond O-2$
- (26) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box A\Box E\Diamond H-2 \rightarrow \Box E\Box M\Diamond O-3 \rightarrow \Box A\Box M\Diamond I-3$
 $\rightarrow \Box M\Box A\Diamond I-3$
- (27) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box E\Box AH-2 \rightarrow \Box E\Box AH-1 \rightarrow \Box A\Box AS-1 \rightarrow \Box A\Box A\Diamond S-1$
- (28) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box E\Box AH-2 \rightarrow \Box E\Box AH-1 \rightarrow \Box A\Box AS-1 \rightarrow \Box A\Box A\Diamond S-1$
 $\rightarrow \Box A\Box F\Diamond O-2$
- (29) $\Box EM\Diamond O-3 \rightarrow \Box A\Box EH-2 \rightarrow \Box E\Box AH-2 \rightarrow \Box E\Box AH-1 \rightarrow \Box A\Box AS-1 \rightarrow \Box A\Box A\Diamond S-1$
 $\rightarrow \Box F\Box A\Diamond O-3$

Proof:

- [1] $\vdash \Box no(v, w) \wedge most(v, z) \rightarrow \Diamond not\ all(z, w)$ (i.e. $\Box EM\Diamond O-3$, Theorem 1)
- [2] $\vdash \Box no(w, v) \wedge most(v, z) \rightarrow \Diamond not\ all(z, w)$ (i.e. $\Box EM\Diamond O-4$, by [1] and Fact (4.2))
- [3] $\vdash \neg \Diamond not\ all(z, w) \wedge \Box no(v, w) \rightarrow \neg most(v, z)$ (by [1] and Rule 2)
- [4] $\vdash \Box \neg not\ all(z, w) \wedge \Box no(v, w) \rightarrow \neg most(v, z)$ (by [3] and Fact (3.2))
- [5] $\vdash \Box all(z, w) \wedge \Box no(v, w) \rightarrow at\ most\ half\ of\ the(v, z)$
 (i.e. $\Box A\Box EH-2$, by [4], Fact (2.1) and Fact (2.5))

- [6] $\vdash \neg \diamond \text{not all}(z, w) \wedge \text{most}(v, z) \rightarrow \neg \square \text{no}(v, w)$ (by [1] and Rule 2)
- [7] $\vdash \square \neg \text{not all}(z, w) \wedge \text{most}(v, z) \rightarrow \diamond \neg \text{no}(v, w)$ (by [6], Fact (3.1) and Fact (3.2))
- [8] $\vdash \square \text{all}(z, w) \wedge \text{most}(v, z) \rightarrow \diamond \text{some}(v, w)$
(i.e. $\square \text{AM} \diamond \text{I-1}$, by [7], Fact (2.1) and Fact (2.3))
- [9] $\vdash \square \text{all} \neg(v, w) \wedge \text{most}(v, z) \rightarrow \diamond \text{some} \neg(z, w)$ (by [1], Fact (1.2) and Fact (1.4))
- [10] $\vdash \square \text{all}(v, D-w) \wedge \text{most}(v, z) \rightarrow \diamond \text{some}(z, D-w)$ (i.e. $\square \text{AM} \diamond \text{I-3}$, by [9] and Definition 2)
- [11] $\vdash \neg \diamond \text{not all}(z, w) \wedge \square \text{no}(w, v) \rightarrow \neg \text{most}(v, z)$ (by [2] and Rule 2)
- [12] $\vdash \square \neg \text{not all}(z, w) \wedge \square \text{no}(w, v) \rightarrow \neg \text{most}(v, z)$ (by [11] and Fact (3.2))
- [13] $\vdash \square \text{all}(z, w) \wedge \square \text{no}(w, v) \rightarrow \text{at most half of the}(v, z)$
(i.e. $\square \text{A} \square \text{EH-4}$, by [12], Fact (2.1) and Fact (2.5))
- [14] $\vdash \neg \diamond \text{not all}(z, w) \wedge \text{most}(v, z) \rightarrow \neg \square \text{no}(w, v)$ (by [2] and Rule 2)
- [15] $\vdash \square \neg \text{not all}(z, w) \wedge \text{most}(v, z) \rightarrow \diamond \neg \text{no}(w, v)$ (by [14], Fact (3.1) and Fact (3.2))
- [16] $\vdash \square \text{all}(z, w) \wedge \text{most}(v, z) \rightarrow \diamond \text{some}(w, v)$
(i.e. $\text{M} \square \text{A} \diamond \text{I-4}$, by [15], Fact (2.1) and Fact (2.3))
- [17] $\vdash \square \text{no} \neg(z, w) \wedge \square \text{all} \neg(v, w) \rightarrow \text{at most half of the}(v, z)$
(by [5], Fact (1.1) and Fact (1.2))
- [18] $\vdash \square \text{no}(z, D-w) \wedge \square \text{all}(v, D-w) \rightarrow \text{at most half of the}(v, z)$
(i.e. $\square \text{E} \square \text{AH-2}$, by [17] and Definition 2)
- [19] $\vdash \square \text{no} \neg(z, w) \wedge \text{most}(v, z) \rightarrow \diamond \text{not all} \neg(v, w)$ (by [8], Fact (1.1) and Fact (1.3))
- [20] $\vdash \square \text{no}(z, D-w) \wedge \text{most}(v, z) \rightarrow \diamond \text{not all}(v, D-w)$
(i.e. $\square \text{EM} \diamond \text{O-1}$, by [19] and Definition 2)
- [21] $\vdash \square \text{all}(v, D-w) \wedge \text{most}(v, z) \rightarrow \diamond \text{some}(D-w, z)$ (i.e. $\text{M} \square \text{A} \diamond \text{I-3}$, by [10] and Fact (4.1))
- [22] $\vdash \square \text{no}(D-w, z) \wedge \square \text{all}(v, D-w) \rightarrow \text{at most half of the}(v, z)$
(i.e. $\square \text{E} \square \text{AH-1}$, by [18] and Fact (4.2))
- [23] $\vdash \square \text{no}(D-w, z) \wedge \text{most}(v, z) \rightarrow \diamond \text{not all}(v, D-w)$
(i.e. $\square \text{EM} \diamond \text{O-2}$, by [20] and Fact (4.2))
- [24] $\vdash \square \text{all}(v, D-w) \wedge \text{fewer than half of the} \neg(v, z) \rightarrow \diamond \text{not all} \neg(D-w, z)$
(by [21], Fact (1.6) and Fact (1.3))
- [25] $\vdash \square \text{all}(v, D-w) \wedge \text{fewer than half of the}(v, D-z) \rightarrow \diamond \text{not all}(D-w, D-z)$
(i.e. $\text{F} \square \text{A} \diamond \text{O-3}$, by [24] and Definition 2)
- [26] $\vdash \square \text{all} \neg(D-w, z) \wedge \square \text{all}(v, D-w) \rightarrow \text{at least half of the} \neg(v, z)$
(by [22], Fact (1.2) and Fact (1.7))

- [27] $\vdash \Box all(D-w, D-z) \wedge \Box all(v, D-w) \rightarrow at\ least\ half\ of\ the(v, D-z)$
(i.e. $\Box A \Box AS-1$, by [26] and Definition 2)
- [28] $\vdash \Box all \neg(D-w, z) \wedge fewer\ than\ half\ of\ the \neg(v, z) \rightarrow \Diamond not\ all(v, D-w)$
(by [23], Fact (1.2) and Fact (1.6))
- [29] $\vdash \Box all(D-w, D-z) \wedge fewer\ than\ half\ of\ the(v, D-z) \rightarrow \Diamond not\ all(v, D-w)$
(i.e. $\Box AF \Diamond O-2$, by [28] and Definition 2)
- [30] $\vdash \Box all(z, w) \wedge \Box no(v, w) \rightarrow \Diamond at\ most\ half\ of\ the(v, z)$
(i.e. $\Box A \Box E \Diamond H-2$, by [5], Fact (5.3) and Rule 1)
- [31] $\vdash \Box all(z, w) \wedge \Box no(w, v) \rightarrow \Diamond at\ most\ half\ of\ the(v, z)$
(i.e. $\Box A \Box E \Diamond H-4$, by [30] and Fact (4.2))
- [32] $\vdash \neg \Diamond at\ most\ half\ of\ the(v, z) \wedge \Box all(z, w) \rightarrow \neg \Box no(v, w)$ (by [30] and Rule 2)
- [33] $\vdash \Box \neg at\ most\ half\ of\ the(v, z) \wedge \Box all(z, w) \rightarrow \Diamond \neg no(v, w)$
(by [32], Fact (3.1) and Fact (3.2))
- [34] $\vdash \Box most(v, z) \wedge \Box all(z, w) \rightarrow \Diamond some(v, w)$
(i.e. $\Box A \Box M \Diamond I-1$, by [33], Fact (2.6) and Fact (2.3))
- [35] $\vdash \neg \Diamond at\ most\ half\ of\ the(v, z) \wedge \Box no(v, w) \rightarrow \neg \Box all(z, w)$ (by [30] and Rule 2)
- [36] $\vdash \Box \neg at\ most\ half\ of\ the(v, z) \wedge \Box no(v, w) \rightarrow \Diamond \neg all(z, w)$
(by [35], Fact (3.1) and Fact (3.2))
- [37] $\vdash \Box most(v, z) \wedge \Box no(v, w) \rightarrow \Diamond not\ all(z, w)$
(i.e. $\Box E \Box M \Diamond O-3$, by [36], Fact (2.6) and Fact (2.2))
- [38] $\vdash \Box no \neg(z, w) \wedge \Box all \neg(v, w) \rightarrow \Diamond at\ most\ half\ of\ the(v, z)$
(by [30], Fact (1.1) and Fact (1.2))
- [39] $\vdash \Box no(z, D-w) \wedge \Box all(v, D-w) \rightarrow \Diamond at\ most\ half\ of\ the(v, z)$
(i.e. $\Box E \Box A \Diamond H-2$, by [38] and Definition 2)
- [40] $\vdash \neg \Diamond at\ most\ half\ of\ the(v, z) \wedge \Box all(z, w) \rightarrow \neg \Box no(w, v)$ (by [31] and Rule 2)
- [41] $\vdash \Box \neg at\ most\ half\ of\ the(v, z) \wedge \Box all(z, w) \rightarrow \Diamond \neg no(w, v)$
(by [40], Fact (3.1) and Fact (3.2))
- [42] $\vdash \Box most(v, z) \wedge \Box all(z, w) \rightarrow \Diamond some(w, v)$
(i.e. $\Box M \Box A \Diamond I-4$, by [41], Fact (2.6) and Fact (2.3))
- [43] $\vdash \neg \Diamond at\ most\ half\ of\ the(v, z) \wedge \Box no(w, v) \rightarrow \neg \Box all(z, w)$ (by [31] and Rule 2)
- [44] $\vdash \Box \neg at\ most\ half\ of\ the(v, z) \wedge \Box no(w, v) \rightarrow \Diamond \neg all(z, w)$ (by [43], Fact (3.1) and Fact (3.2))
- [45] $\vdash \Box most(v, z) \wedge \Box no(w, v) \rightarrow \Diamond not\ all(z, w)$

(i.e. $\Box E\Box M\Diamond O-4$, by [44], Fact (2.6) and Fact (2.2))

[46] $\vdash \Box most(v, z) \wedge \Box no\bar{\neg}(z, w) \rightarrow \Diamond not\ all\bar{\neg}(v, w)$ (by [34], Fact (1.1) and Fact (1.3))

[47] $\vdash \Box most(v, z) \wedge \Box no(z, D-w) \rightarrow \Diamond not\ all(v, D-w)$

(i.e. $\Box E\Box M\Diamond O-1$, by [46] and Definition 2)

[48] $\vdash \Box most(v, z) \wedge \Box all\bar{\neg}(v, w) \rightarrow \Diamond some\bar{\neg}(z, w)$ (by [37], Fact (1.2) and Fact (1.4))

[49] $\vdash \Box most(v, z) \wedge \Box all(v, D-w) \rightarrow \Diamond some(z, D-w)$

(i.e. $\Box A\Box M\Diamond I-3$, by [48] and Definition 2)

[50] $\vdash \Box no(D-w, z) \wedge \Box all(v, D-w) \rightarrow \Diamond at\ most\ half\ of\ the(v, z)$

(i.e. $\Box E\Box A\Diamond H-1$, by [39] and Fact (4.2))

[51] $\vdash \Box most(v, z) \wedge \Box no(D-w, z) \rightarrow \Diamond not\ all(v, D-w)$

(i.e. $\Box E\Box M\Diamond O-2$, by [47] and Fact (4.2))

[52] $\vdash \Box most(v, z) \wedge \Box all(v, D-w) \rightarrow \Diamond some(D-w, z)$

(i.e. $\Box M\Box A\Diamond I-3$, by [49] and Fact (4.1))

[53] $\vdash \Box all(D-w, D-z) \wedge \Box all(v, D-w) \rightarrow \Diamond at\ least\ half\ of\ the(v, D-z)$

(i.e. $\Box A\Box A\Diamond S-1$, by [27], Fact (5.3) and Rule 1)

[54] $\vdash \bar{\neg}\Diamond at\ least\ half\ of\ the(v, D-z) \wedge \Box all(D-w, D-z) \rightarrow \bar{\neg}\Box all(v, D-w)$

(by [53] and Rule 2)

[55] $\vdash \Box \bar{\neg} at\ least\ half\ of\ the(v, D-z) \wedge \Box all(D-w, D-z) \rightarrow \Diamond \bar{\neg} all(v, D-w)$

(by [54], Fact (3.1) and Fact (3.2))

[56] $\vdash \Box fewer\ than\ half\ of\ the(v, D-z) \wedge \Box all(D-w, D-z) \rightarrow \Diamond not\ all(v, D-w)$

(i.e. $\Box A\Box F\Diamond O-2$, by [55], Fact (2.8) and Fact (2.2))

[57] $\vdash \bar{\neg}\Diamond at\ least\ half\ of\ the(v, D-z) \wedge \Box all(v, D-w) \rightarrow \bar{\neg}\Box all(D-w, D-z)$

(by [53] and Rule 2)

[58] $\vdash \Box \bar{\neg} at\ least\ half\ of\ the(v, D-z) \wedge \Box all(v, D-w) \rightarrow \Diamond \bar{\neg} all(D-w, D-z)$

(by [57], Fact (3.1) and Fact (3.2))

[59] $\vdash \Box fewer\ than\ half\ of\ the(v, D-z) \wedge \Box all(v, D-w) \rightarrow \Diamond not\ all(D-w, D-z)$

(i.e. $\Box F\Box A\Diamond O-3$, by [58], Fact (2.8) and Fact (2.2))

Now, the other 29 generalized modal syllogisms have been deduced from the validity of $\Box EM\Diamond O-3$. Similarly, more valid syllogisms can be inferred from it. This indicates that there are reducible relations between/among these syllogisms. Their validity can be proven similar to Theorem 1.

V. CONCLUSION

Due to the large number of generalized quantifiers in the English language, this paper only studies the fragment of generalized modal syllogistic that contains the quantifiers in Square-*all* and Square-*most*. This paper proves that there are reducible relations between/ among the generalized modal syllogism $\Box EM \Diamond O-3$ and at least the above 29 valid generalized modal syllogisms. To be specific, this paper firstly proves the validity of $\Box EM \Diamond O-3$ on the basis of generalized quantifier theory, possible-world semantics, and set theory. Then, according to some facts and inference rules, the above 29 valid generalized modal syllogisms are derived from $\Box EM \Diamond O-3$.

This method can also be used to study syllogisms with other generalized quantifiers, such as *at most 1/3 of the*, *more than 1/3 of the*, *at least 2/3 of the*, *fewer than 2/3 of the*. It is obvious that the above results obtained by deduction have not only consistency, but also theoretical value for the development of inference theory in artificial intelligence.

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REFERENCES RÉFÉRENCES REFERENCIAS

1. J. Łukasiewicz, (1957). Aristotle's Syllogistic: From the Standpoint of Modern Formal Logic, second edition, Oxford: Clarendon Press.
2. G. Patzig, (1969). Aristotle's Theory of the Syllogism. D. Reidel.
3. P. Halmos, (1974). Naive Set Theory, New York: Springer-Verlag.
4. F. Chellas, (1980). Modal Logic: an Introduction, Cambridge: Cambridge University Press.
5. F. Johnson, (2004). Aristotle's modal syllogisms, Handbook of the History of Logic, I, pp. 247-338.
6. P. Murinová and V. Novák, (2012). A Formal Theory of Generalized Intermediate Syllogisms, Fuzzy Sets and Systems, 186, pp. 47-80.
7. J. Endrullis and L. Moss, (2015). Syllogistic logic with 'Most', in V. de Paiva et al. (eds.), Logic, Language, Information, and Computation, pp. 124-139.
8. Xiaojun Zhang, Baoxiang Wu, (2021). Research on Chinese Textual Reasoning, Beijing: People's Publishing House. (in Chinese)
9. Long Wei, (2023). Formal system of categorical syllogistic logic based on the syllogism AEE-4. Open Journal of Philosophy, (13), 97-103.
10. Hui Li, (2023). Reduction between categorical syllogisms based on the syllogism EIO-2. Applied Science and Innovative Research, (7), 30-37.
11. Cheng Zhang and Xiaojun Zhang (2023). Reduction between Aristotelian Modal Syllogisms Based on the Syllogism $\Diamond IA \Diamond I-3$. Open Journal of Philosophy, 2023, 13, 145-154.
12. Jing Xu and Xiaojun Zhang (2023). The Reducibility of Generalized Modal Syllogisms Based on $\Box AM \Diamond I-1$. SCIREA Journal of Philosophy, Volume 3, Issue 1.